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# When do redundant fluidic networks outperform non-redundant ones?

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received 9 December 2016; accepted in final form 25 April 2017

published online 12 May 2017

PACS 47.60.Dx – Flows in ducts and channels

PACS 47.63.-b – Biological fluid dynamics

PACS 89.20.-a – Interdisciplinary applications of physics

**Abstract** – Redundancy constitutes a fundamental and intrinsic aspect of healthy vasculatures. Built-in redundancy might also be a desirable feature in man-made microfluidic devices. We show that redundant and non-redundant networks, built to have identical resistances to flow when unobstructed, allow for very different flows when they are occluded; redundant ones —densely occluded at a certain bifurcation level— allowing for larger flows than non-redundant ones —obstructed above relatively small thresholds. We also show that redundancy protects vessels against the large shear-rate gradients that occlusions would cause if it were not present. Our study allows one to quantify a network tolerance against blockage, provides guidance in the tailoring of microfluidic devices, and offers novel insights into why nature has selected intrinsic redundancy over thicker vessels to assure blood supply at key places of the organisms.

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**Introduction.** – Redundancy in natural processes is present in a wide variety of systems, ranging from metabolism, ecosystems and genetics [1–4]. Redundancy provides alternative ways for a process to occur. It has a protective function in case of failure of fragile parts or mechanisms. Redundancy is energetically expensive in general, however natural systems present redundant features whenever their benefits outweigh their costs. In imitation of nature, man-made systems have built-in redundancy to assure reliability when the cost of failure is too high. Redundancy for instance is used in the semiconductor industry to make reliable microchips [5,6]. It is present in the hardware of computers and in the software of operating systems [7–9].

Redundancy in fluidic networks occurs in many natural systems, for example in the xylem of trees, that transport the sap, or in the lymphatic vessel network of mammals. For mammals, the degree of intrinsic redundancy in blood vessel networks is particularly relevant in vascular

diseases [10–13]. Natural bypasses in the coronary arteries that irrigate the heart, can save lives in patients that are lucky to have enough of them [10]. Redundancy also constitutes an intrinsic aspect of healthy cerebrovascular circulation. Its role is clearly to assure the preservation of blood flow, that maintains the delivery of nutrients to a tissue, by providing alternative routes in the presence of obstructions. It plays a key role in the physiology of cerebral ischemia (lack of oxygen) [14–21].

Detailed simulations of flow around partial vascular obstructions in redundant structures have been reported [22–26]. These simulations describe the velocity, the existence of vortices and the value of the wall shear stress at different locations within the vessels.

In this paper we are interested in the global effects that built-in —or intrinsic— redundancy has on the total flow through a network. We study generic fluidic networks, potentially useful in microfluidics —where obstructions can cause the failure of a chip— and realistic animal vasculatures —where occlusions can compromise tissue irrigation. For animal vasculatures, there is another type of

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redundancy, in which organisms create more vessels as a response to an unhealthy condition. We differentiate this situation from the one of our studies, which we call intrinsic, for which redundancy is present in healthy vasculatures. Despite being intuitive that intrinsic redundancy makes robust networks, one can envision other network topologies having identical resistances to flow, like networks with wider vessels. So the question arises on why and when it is better to have redundant networks. We explain in which situations the flow assured by intrinsic redundancy is more effective than the one assured by non-redundant networks. We propose a criterion to measure the tolerance of networks against occlusions. We explore the not so intuitive issue on where in the network redundancy has a larger benefit when obstructions are present. We also study the impact that redundancy has on the shear rate across the network, since shear rate is a crucial factor of failure in materials, and it is a pivotal agent in remodelling arterial vasculatures. We show that redundancy prevents the enormous shear gradients that obstructions would cause if it were not present. Our results are not only a step further in the understanding of intrinsic redundancy, they provide novel insights into why, despite an equal energetic flow cost, nature has selected intrinsic redundancy over equivalent non-redundant networks to assure blood supply for tissue irrigation at key places of the organism. Our studies also provide guidance in tailoring robust networks in microfluidics, in which an adequate choice of the amount and places where redundancy appears can prevent failure of the network by blockage of particles and bubbles.

**Model of fluidic networks.** – Tree-like networks are ubiquitous in natural systems [27–32]. They also represent ideal mathematical models to study networks [30–33] and have been used to study microfluidic devices [34]. We therefore take them as model systems whose results could be applied in different fields.

A non-redundant tree-like network is shown in red in fig. 1. By the addition of bypasses as in [35] (shown in green in fig. 1), redundancy is introduced in the non-redundant underlying tree-like network. The bypasses have the same dimensions of the circumvented levels. Obstructions in half of the conduits at a certain bifurcation level, are represented by crosses, and lead to a local reduction in the cross-sectional area. This choice for obstructing the network, that does not represent how natural networks get occluded, has been chosen in order to highlight the role of the topological place,  $n$ , in which occlusions occur; this one has been used previously in non-redundant networks, to relate the total flow through them, with the place in which obstructions occur [36].

In order to have a generic model system, useful for instance, in the design of microfluidic devices, we study networks of equal conduits. With the purpose of studying realistic physiological networks, such as the circulatory system, or the water transport system in plants [27],

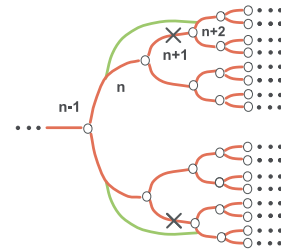


Fig. 1: (Colour online) Illustration of a tree-like network with redundancy in the form of bypasses that bridge levels  $n$  and  $n + 2$ , and obstructions present in conduits at level  $n + 1$ .

we consider networks in which vessels radii follow Murray’s law [33]. This one relates the parent vessel radius,  $r_p$ , and the two child vessels radii after a bifurcation,  $r_{ch1}$  and  $r_{ch2}$ , as:  $r_p^3 = r_{ch1}^3 + r_{ch2}^3$ . Physiological studies show that a large part of the mammalian vasculatures have a reasonable agreement with this law [28,29]. There is also interest in constructing biomimetic micro and nanofluidic devices that follow the optimization principles which lead to Murray’s law in biological networks [34]. We use an electrical analogy in which the resistance  $R$  of each vessel is given by  $R = \frac{l}{AK}$ , where  $l$ ,  $A$  and  $K$  are the conduit’s length, cross-sectional area and dynamic permeability<sup>1</sup>, respectively. The dynamic permeability is a function of frequency and contains information about the fluid and the confining media. For our studies, we use the simplest model for a linear viscoelastic fluid flowing inside rigid tubular structures, in order to comprise both, complex fluids such as blood, and Newtonian liquids, such as water. The permeability for a single tube of radius  $r$ , is given by  $K(\omega) = \frac{in}{\omega\rho} \left[ 1 - \frac{2J_1(\beta r)}{\beta r J_0(\beta r)} \right]$  [37]. Here  $J_0$  and  $J_1$  are Bessel functions of order zero and one, respectively,  $\beta^2 = \frac{\rho}{\eta} (t_r \omega^2 + i\omega)$ ;  $t_r$  and  $\eta$  are, respectively, the relaxation time and the viscosity of the fluid. For a Newtonian fluid  $t_r = 0$ . For the obstructed conduits the cross-sectional area is  $A^{obs} = A(1-f)$ , where  $f$  is the obstructed fraction of the tube’s cross-section, whose radius becomes  $r^{obs} = r\sqrt{1-f}$ , where  $r$  is the unobstructed radius. The resistance of the obstructed tube is  $R^{obs} = \frac{l}{A^{obs}K^{obs}}$  where  $K^{obs} = K(r = r^{obs})$ . We take into account flow conservation and consider that the total pressure drop,  $\Delta p$ , is equal to the sum of individual pressure drops. The system of equations obtained from these considerations are solved to give the network response  $\chi$ , defined as the total length divided by the network resistance. For systems driven by an imposed time-dependent pressure drop, the time-dependent total flow,  $Q(t)$ , through a network of length  $L$ , is given by [38]

$$Q(t) = -\frac{1}{2\pi\eta L} \int_{-\infty}^{\infty} \Delta p(t - \tau) \left[ \int_{-\infty}^{\infty} \chi(\omega) e^{-i\omega\tau} d\omega \right] d\tau. \quad (1)$$

<sup>1</sup>The name permeability has been borrowed from the area of porous media; and has been extended, two decades ago, to account for the joint dynamic response of the system fluid-confining media.

This can be seen in frequency domain as  $\hat{Q}(\omega) = -\frac{1}{\eta L}\chi(\omega)\Delta\hat{p}(\omega)$ . For the particular case of a single-mode pulsatile pressure drop  $\Delta p(t) = \Delta p_0 \cos(\omega_0 t)$ , the flow is given by [39]

$$Q(t) = -\frac{1}{\eta} [Re\chi(\omega_0) \cos(\omega_0 t) + Im\chi(\omega_0) \sin(\omega_0 t)] \frac{\Delta p_0}{L}. \quad (2)$$

Therefore, knowledge of the response function permits the calculation of the time-dependent network flow. The real and imaginary parts of the response give the contributions in phase and in quadrature between the flow and the pressure drop, respectively. The maximum flow amplitude is given by

$$|Q(t)|_{max} = \frac{1}{\eta} |\chi(\omega_0)| \frac{\Delta p_0}{L}. \quad (3)$$

This shows that the magnitude of the response,  $\chi(\omega_0)$ , is linearly related with the maximum flow amplitude for systems driven by single-mode pressure drops<sup>2</sup>. For dynamic pressure drops consisting of several modes, the linear character of the equations allow to express flow as a sum of all modes involved in the pressure drop, each one weighted by its correspondent response<sup>3</sup>.

**Redundant vs. non-redundant networks.** – We compare redundant with non-redundant vascular networks that in the absence of obstructions oppose the same resistance to flow. We call these ones equivalent networks. For them, flow, at a given pressure drop, has exactly the same energetic cost per unit time. To built them, we choose a non-redundant network and construct the equivalent redundant network by adjusting its radii to thinner radii, in such a way that both responses (and maximum flow amplitudes) are equal at a particular chosen frequency. Figures 2(a) and (b) illustrate these equivalent networks. Since in microfluidics, fluid flow is generally steady, we choose zero frequency for networks of equal conduits. Flows in natural systems, such as in vascular networks, are pulsed with characteristic frequencies

<sup>2</sup>For the particular case of a constant pressure drop,  $\Delta p_0$ , the relation between flow and pressure drop for a single tube of radius,  $r$ , and length,  $l$ , is given by  $Q = -\frac{1}{\eta} AK(0) \frac{\Delta p_0}{l}$ , with  $K(0) = \frac{r^2}{8}$ . This expression can be written as the well-known steady-state Darcy's law for a single tube  $Q = -\frac{\pi r^4}{8\eta} \frac{\Delta p_0}{l}$  for Poiseuille flow. It can also be written in terms of the resistance of the tube,  $R = \frac{8l}{\pi r^4}$ , as  $Q = -\frac{1}{R\eta} \Delta p_0$ . The analogous expression for a network, comes from eq. (2) when  $\omega_0 = 0$ , that is,  $Q = -\frac{1}{\eta} \chi(0) \frac{\Delta p_0}{L}$ , where  $\chi$ , by definition, is the total length divided by the network resistance, calculated using the electrical analogy with the time-independent resistances of individual vessels.

<sup>3</sup>For flows through networks with an imposed flow, the pressure drop is linearly related to it in frequency domain as  $\Delta\hat{p}(\omega) = -\eta L \frac{1}{\chi(\omega)} \hat{Q}(\omega)$ . In that case,  $1/\chi(\omega)$ , can be considered to be the system's response. Expressions to obtain the pressure drop in terms of the imposed flow in time domain, equivalent to equations 1, 2 and 3 can be easily obtained. In that case, the pressure drop would be inversely proportional to  $|\chi(\omega_0)|$ , meaning that large values of  $|\chi(\omega_0)|$  would lead to situations in which the work needed to maintain the imposed flow, against frictional forces, is small. Note that  $\chi$  is independent of the way in which the fluid is driven.

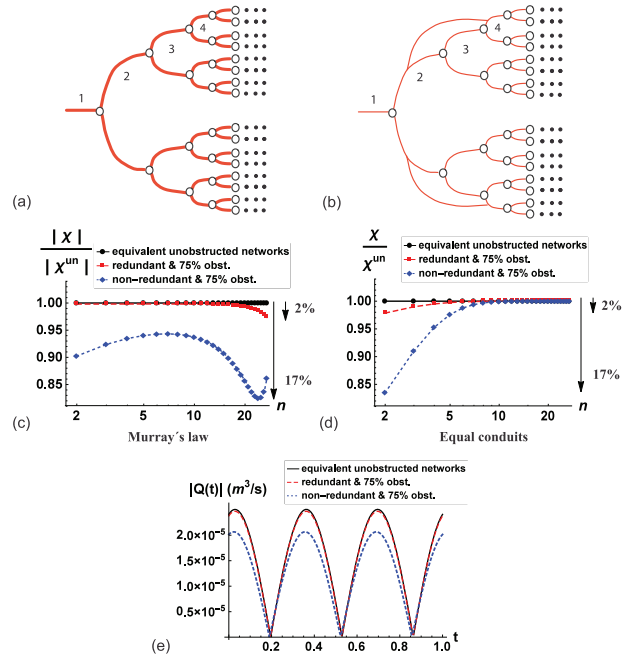


Fig. 2: (Colour online) Illustration of equivalent non-redundant (a) and redundant (b) networks that transport equal flow at a chosen frequency in the absence of obstructions. Responses of (c) blood in networks that follows Murray's law, and (d) water in networks of equal conduits as a function of the level  $n$  where the obstructions are introduced. In both figures, the response of the equivalent networks —continuous line with circles— has been chosen as a normalization factor and is shown for reference. In the presence of obstructions redundant and non-redundant networks are no longer equivalent. The responses of non-redundant networks —dotted lines with rhombi— decrease by approximately 17% for both, Murray networks (c) and equal conduits networks (d). In contrast, the responses of redundant networks, in the presence of obstructions —dashed line with squares— have approximately the value of the unobstructed equivalent networks (c) and (d). (e) One-mode time-dependent blood flow, as a function of time, in a redundant —dashed line— and a non-redundant —dotted line— network that follows Murray's law, occluded at level  $n = 25$ . Flow in the unobstructed equivalent networks is shown with continuous line for reference. A realistic amplitude for the pressure drop is set to 8.5 kPa. For (c) and (e) the radius of the first vessel of the non-redundant network corresponds to the one of the dog's aorta. The vessels lengths follow a power law to fit the data for the dog circulatory system. The frequency has been chosen as 1.5 Hz, the rest heart rate of the dog. The blood parameters were taken as:  $\rho = 1050 \text{ kg/m}^3$ ,  $\eta = 15 \times 10^{-3} \text{ Pa s}$  and  $t_r = 1 \times 10^{-3} \text{ s}$ . For (d) equal conduits of radius  $r = 10 \mu\text{m}$  and length  $l = 2.0 \text{ mm}$  have been taken. Computations were made at zero frequency. The water parameters were taken as:  $\rho = 1000 \text{ kg/m}^3$  and  $\eta = 1 \times 10^{-3} \text{ Pa s}$ .

dependent on processes and species. So, for networks that follow Murray's law we work at a finite frequency (for the figure, we have chosen the resting heart rate of the dog). Figures 2(c) and (d) indicate how the networks' response functions behave as a function of the topological index  $n$  in which variations (obstructions, bypasses or both) occur.

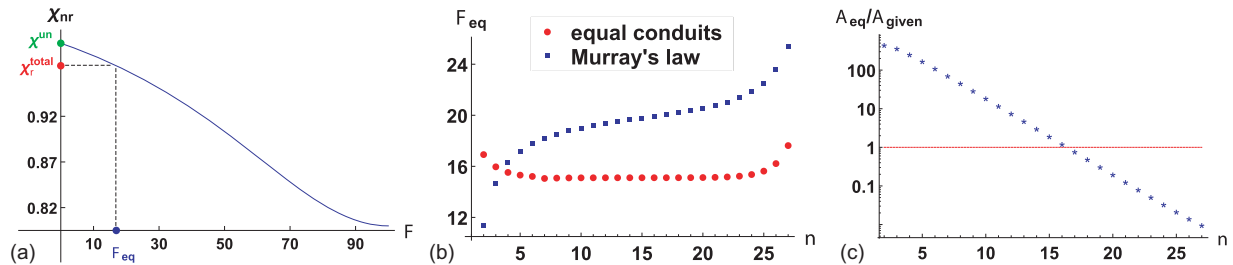


Fig. 3: (Colour online)  $F_{eq}$ , a criterion to measure the robustness of a redundant network. The lower the value of  $F_{eq}$ , the larger the advantage redundancy gives at a given  $n$ . (a) Network response of an obstructed non-redundant network,  $\chi_{nr}$ , as a function of the degree of obstruction,  $F$ . For this example, obstructions of 17% ( $F_{eq} = 17$ ) at level  $n = 3$  in the non-redundant network cause as much flow shortage as total obstructions do in redundant networks. In the ordinate axis, the response of the unobstructed equivalent networks,  $\chi^{un}$ , chosen as a normalization factor, is marked; and the response of the redundant network with occlusions of 100% in area (in the vessels skipped by the bypasses),  $\chi_r^{total}$ , is also indicated for reference. For this example, a redundant network is more convenient than an equivalent thicker vessels network for obstructions larger than 17%. (b)  $F_{eq}$  is shown for different values of the topological index  $n$ , for networks that follow Murray's law and for networks of equal conduits. (c)  $A_{eq}/A_{given}$  for Murray's networks. When the quantity  $A_{eq}/A_{given} < 1$ , redundant networks outperform non-redundant ones. For this example, the normalization factor,  $A_{given}$ , has been chosen as the area of the vessel at level  $n = 21$ , which results in redundancy being preferable for all levels above  $n = 16$ . Parameters are as in fig. 2

In each of the figures, the continuous lines with circles indicate the response of the two initially equivalent networks (their response is identical by construction), and has been chosen as a normalization factor for all the curves. We compare how non-redundant and redundant networks respond to the presence of obstructions in the cross-sectional area. Each of the figures show that the equivalence is lost when the networks are occluded: the response of non-redundant networks —dotted lines with rhombi— decreases by approximately 17% for certain levels for both Murray's networks (fig. 2(c)) and equal vessels networks (fig. 2(d)). In contrast, the response of the redundant networks in the presence of obstructions —dashed lines with squares— is effectively equal to the unobstructed equivalent networks showing a maximum decrease of 2% in both types of networks.

The particular way in which the response of obstructed networks decreases, in a non-monotonic way (as a function of the level in which obstructions occur) for networks that follow Murray's law (fig. 2(c)) and in a monotonic way for networks of equal conduits (fig. 2(d)), is due to the specific sequence of resistances between subsequent levels. This is not the subject of this manuscript, and has been reported elsewhere [36] for non-redundant networks.

Figure 2(e) illustrates the impact that differences in the network response at a certain level  $n$ , have on the total flow through the network, illustrating that flow decrease follows the tendency of the response.

The presence of redundancy is most efficient when the bypasses skip the obstructed sites (as considered here); however we have verified that it represents a benefit for total flow regardless of its position in the network.

These results quantify how, according to fluid dynamics, redundant networks allow for more flow than equivalent non-redundant networks, at a given pressure gradient, when obstructions are present. They agree with common

knowledge and intuition on how intrinsic redundancy works; but emphasize that its advantage, is only manifest and relevant in the presence of obstructions, while for unblocked systems, other type of topologies could be as efficient to transport flow.

#### Measuring the tolerance against occlusions. —

Figure 3(a), shows a curve for the response of a non-redundant network,  $\chi_{nr}$ , as a function of the percentage of obstructed area,  $F = 100f$ , at a given bifurcation level. In the ordinate axis, the response of the unobstructed equivalent networks,  $\chi^{un}$ , chosen as a normalization factor, is marked, and the response of the redundant network with occlusions of 100% in area —in the vessels skipped by the bypasses—  $\chi_r^{total}$ , is also indicated. At relatively small percentages of the obstructed area,  $F_{eq}$ , (17% for the example of equal conduits in fig. 3(a)), the response of a non-redundant network is equal to the response of the redundant network with total occlusions. In terms of flow, this means that areas, in the non-redundant networks, obstructed by more than  $F_{eq}$ , give smaller flows than redundant networks with total occlusions. Figure 3(a) is qualitatively the same for a network that follows Murray's law (not shown). We propose  $F_{eq}$  as a criterion to measure the robustness of a network with intrinsic redundancy since it gives a relative measure of the network's tolerance against occlusions. When the percentage of obstructed area is above the threshold,  $F_{eq}$ , having redundancy is more convenient in terms of flow, than having wider vessels. The lower  $F_{eq}$ , the larger the advantage redundancy gives at a given  $n$ ; the larger the tolerance to occlusions of the equivalent redundant network, and the less tolerant to obstructions the equivalent non-redundant network is. This percentage has been chosen in the *worst case scenario* for which occlusions in equivalent redundant networks are 100% in area.  $F_{eq}$  is different for each topological place  $n$  in which occlusions occur and depends on the structure



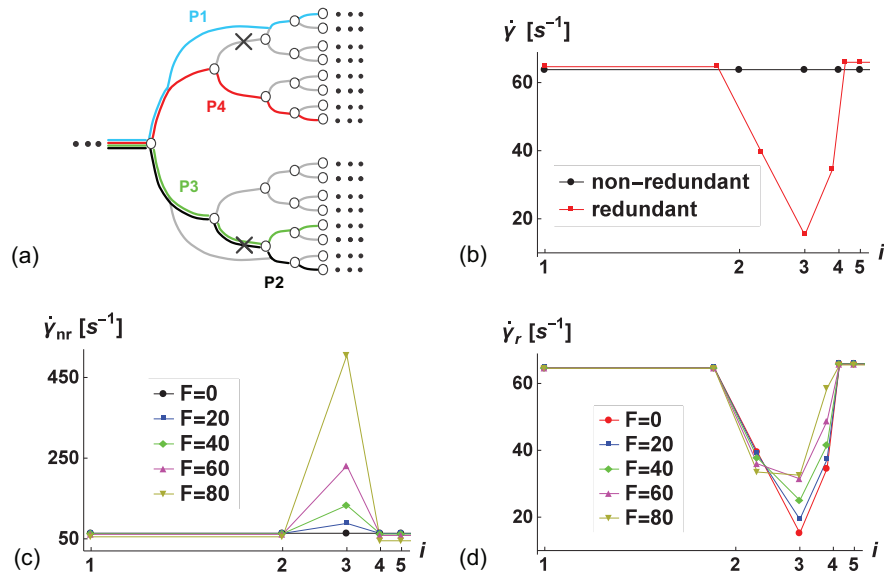


Fig. 4: (Colour online) Shear rate value along different paths in redundant and non-redundant networks. (a) Illustration of the four possible paths for fluid flow. (b) Value of the shear rate along path P2 for a non-redundant network (line with circles) and for a redundant network (line with squares). (c) Value of the shear rate along path P2 in a non-redundant network with obstructions of different degrees. (d) Value of the shear rate along path P2 in a redundant network with obstructions of different degrees. The index  $i$  in (b), (c) and (d) denotes the bifurcation level of the network.

of the underlying network. This is illustrated in fig. 3(b) for a network that follows Murray’s law and for a network of equal conduits.  $F_{eq}$  gives the percentage of obstructed area —of level  $n$ — above which it is better to have redundancy than a network of thicker vessels. For networks of equal vessels the advantage of redundancy is relatively independent of the place in which it occurs. For more complex networks, whose area changes with  $n$ , like Murray’s networks, fig. 3(b) should be interpreted with caution. It is only useful when considering a value of  $n$  at once. In order to interpret,  $F_{eq} = 100f_{eq}$ , across different values of  $n$ , we construct the equivalent area,  $A_{eq} = f_{eq}A_n$ , where,  $A_n$ , is the cross-sectional area of a vessel at level  $n$ . Figure 3(c) shows  $A_{eq}/A_{given}$  for networks that follow Murray’s law. The normalization factor,  $A_{given}$ , is chosen as the area of a vessel at a given level. In front of obstacles of size  $A_{given}$ , in the cross-sectional area, redundancy is preferable for levels whose  $A_{eq}/A_{given}$  is smaller than one. This figure shows that for Murray’s networks the advantage of redundancy is larger when it occurs at internal levels. For networks of equal vessels the curve  $A_{eq}$  is proportional to the curve  $F_{eq}$  (not shown). These results quantify, how advantageous redundancy is at specific sites, and when do redundant networks outperform non-redundant ones.

It is worth mentioning that a global flow decrease in the redundant network due to obstructions, does not prevent the existence of localized regions where the flow actually increases. We observe that rerouting of flow through the bypasses takes place for occlusions of any size; the larger the percentage of obstructed area, the larger the flow increase in the bypasses (not shown).

**Intrinsic redundancy: A shield against large shear gradients.** — The presence of large shear gradients might cause failure of materials or dysfunctional vascular behavior. In vessel networks, large shear rates have been associated with the deposition of platelets in arteries [40] and several studies suggest that large changes in wall shear stress cause the vasculature to be severely altered [41,42]. We study the effect that redundancy has on shear, along the four different possible fluid paths in our model networks. Figure 4(a) indicates these paths: the first one that goes through the bypass (P1, in blue); two paths that go through the obstructed tube (P2, in black and P3, in green; only P2 intersects the bypass downstream after the obstruction); and a path that neither goes through the bypass, nor through the occlusion (P4, in red). For non-redundant networks, the path P1 does not exist, and paths P2 and P3 are equivalent. So for non-redundant networks P2 goes through the obstructed tube and P4 does not.

For a network that follows Murray’s law, by construction, the shear rate is constant in all vessels. Introduction of redundancy introduces gradients in shear rate by decreasing the value of shear, in the four fluid paths, around the place where the bypass is located. Figure 4(b) shows variations of shear rate along path P2 as large as 80%. Although this represents a disadvantage, the panorama changes completely in the presence of obstructions; since in non-redundant networks occlusions cause huge gradients in shear rate. In fig. 4(c) we plot the effect that obstructions have on the shear rate of the vessels along path P2 in a non-redundant network. Obstructions of 80% in area at level  $n = 3$ , cause an increase in shear rate of about

700%. In contrast, the same percentage of obstructions in the redundant network (fig. 4(d)), leads to an increase in shear rate of about 100% “in the correct sense”, meaning that obstructions induce a more uniform shear rate along the flow direction. Redundancy therefore, shields the impact that obstructions have on shear gradients and makes the network less sensitive to the presence of obstructions. An equivalent behavior is obtained along path P3. Paths P1 and P4 present less dramatic changes.

In Nature, redundancy is able to provide fluidic networks a survival advantage due to its ability to accommodate the presence of occlusions in the network, despite introducing shear gradients in the absence of obstructions. This latter, relatively small disadvantage, is part of the cost, in fluidic networks, for having the advantages of built-in redundancy.

For networks of equal conduits, the impact of obstructions in non-redundant networks is weaker, but redundancy still shields the impact that obstructions have on shear gradients (not shown). Figures 4(c) and 4(d) have been calculated for a constant pressure drop across the network. We have verified that the same conclusions are obtained when a constant inflow is imposed (not shown).

**Is there an optimum?** – Introduction of more than one bypass at a certain level, always increases the response. One might therefore wonder why nature does not promote the creation of more redundancy sites, and if there is an optimum number of redundant vessels for a given resistance to flow. Our studies show that the value of the response saturates when the number of parallel bypasses increases (not shown). After saturation, further addition of redundant vessels produces a negligible gain in terms of flow. Regarding shear rate, the addition of more redundant vessels causes slightly larger gradients, which would be somewhat disadvantageous. Our studies indicate that the presence of few redundant vessels gives a large advantage but the addition of a higher number of such vessels, does not represent further gain.

For the case of vascular networks, vessels are formed by living cells, and the extra-vessels would also increase the cost of sustaining the vasculature.

**Conclusions.** – Our results show that the advantage of intrinsic redundancy is only manifest and relevant in the presence of obstructions; while for unblocked fluidic networks, other type of topologies would be as efficient to transport flow. Redundant networks, with obstructions of 100% in half of the vessels at a certain level, outperform non-redundant ones, when the percentage of obstructed area in the later ones is above a relatively small threshold; we study how such a threshold depends on the topological place where obstructions occur, and how it depends on two types of commonly encountered networks. In vascular networks, redundancy ensures blood supply to a tissue even when obstructions, as large as the cross-sectional area, occlude a large proportion of the vessels; while in microfluidic networks, this will ensure the operation of devices in

front of bubbles as wide as the microchannels forming the network.

Redundancy protects networks against the large shear-rate gradients that occlusions cause in non-redundant networks. However, redundancy in turn, introduces relatively small shear gradients that constitute a small disadvantage when the networks are not occluded. Such a small disadvantage is part of the cost of having the enormous advantages, concerning flow and shear rate evenness, when obstructions are present.

Our analysis offers a plausible reason of why nature has selected intrinsic redundancy over thicker vessels to assure blood supply for tissue irrigation at key places of the organism, like in the brain architecture, and why intrinsic redundancy can save lives when it is present in the heart [10]. It also explains why not all vessel networks are redundant, since redundancy entails a cost; both, for the maintenance of vessels, and for the lack of uniformity in shear rate. To pay such cost might be worth it, only when the good performance of a particular network, is crucial for survival. Our studies provide guidance in the design of robust microfluidic devices for which the careful tailoring of built-in redundancy can prevent failure by blockage due to particles or bubbles.

We have worked with an underlying tree-like network that follows Murray’s law because it has proven to be in reasonable agreement with a large part of the mammalian vasculatures. However, at micro scales, there are other types of vasculatures present in mammals, like the tree-dimensional subsurface network of micro vessels connected to the cortical mantle by penetrating arterioles or the profuse vascular networks of interconnected loops, mainly at the arteriole and capillary levels, promoted by tumors [43,44]. For these ones, flow measurements —like the ones reported in [45]— together with a proper definition of an equivalent non-redundant network, would allow to know if the results presented in this work are robust in networks of more complex connectivities.

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We thank ALEJANDRO MEZA ROMERO for contributing to the earlier stages of this work. AT acknowledges financial support from CONACYT (Mexico) through fellowship 245675. IP acknowledges support from MINECO (Spain), Project FIS2015-67837-P, DURSI Project 2014SGR-922, and Generalitat de Catalunya under Program “ICREA Acadèmia”. RDMT acknowledges financial support from Fundos FEDER through COMPETE program and by FCT through project FCOMP-01-0124-FEDER-015708. RDMT acknowledges support from FCT through the Investigador FCT program. ECP declares that the research leading to these results has received funding from the European Union Seventh Framework Programme (FP7-PEOPLE-2011-IIF) under grant agreement No. 301214, as well as financial support from CONACYT project 219584 and PAIP program 5000-9011.

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